

Reheating After Quintessential Inflation and Gravitational Waves

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We investigate the dependence of the gravitational wave spectrum from quintessential inflation on the reheating process. We consider two extreme reheating processes. One is the gravitational reheating by particle creation in the expanding universe in which the beginning of the radiation dominated epoch is delayed due to the presence of the epoch of domination of the kinetic energy of the inflaton (kination). The other is the instant preheating considered by Felder et al. in which the Universe becomes radiation dominated soon after the end of inflation. We find that the spectrum of the gravitational waves at ~ 100 MHz is quite sensitive to the reheating process. This result is not limited to quintessential inflation but applicable to various inflation models. Conversely, the detection or non-detection of primordial gravitational waves at ~ 100 MHz would provide useful information regarding the reheating process in inflation.

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I. INTRODUCTION

Inflationary paradigm solves various problems in the standard big-bang model (flatness, horizon, monopole problems, etc.) and predicts density perturbation and gravitational wave spectra which are both almost scale-invariant. The two of three major predictions of inflation (flat universe, scale-invariant density perturbations) are consistent with the current observations, and the detection of primordial gravitational waves is a litmus test of the inflationary paradigm. The dynamics of inflation is believed to be described by a scalar field called “inflaton” (or multiple of it) with the energy scale of $\sim 10^{15}\text{GeV}$.

On the other hand, recent observations indicate that the present Universe is dominated by dark energy: an (approximately) smooth energy component with negative pressure. If dark energy is dynamical, its dynamics is likely to be described by an ultra-light scalar field with mass $\lesssim H_0$, called “quintessence”.

Then, it is natural to imagine whether it is possible to unify the inflaton and quintessential fields. A toy model was proposed by Peebles and Vilenkin in which the potential consists of two parts: $\lambda\phi^4 + M^4$ ($\phi \leq 0$) for inflation and $\lambda M^8/(\phi^4 + M^4)$ ($\phi \geq 0$) for quintessence [1]. The two potentials are glued at $\phi = 0$ by hand. In this paper, although not essential, as a slightly elaborated model, we consider a smooth, exponential type potential that describes both the inflaton and quintessential parts¹ in the hope that the model could be embedded in particle physics framework. For other models of quintessential inflation, see [2].

In order to recover the hot big-bang universe, the universe has to be reheated after inflation. In models of quintessential inflation proposed so far, the only mechanism of reheating that has been considered is the gravitational particle production in an expanding universe. The spectrum of gravitational waves based on the gravitational reheating was calculated [3, 4, 5] and a strong enhancement at $\sim\text{GHz}$ was found (see also [6] in different context). However, the gravitational reheating process is very inefficient and may lead to cosmological problems associated with overproduction of dangerous relics such as gravitinos and moduli [7]. Recently, Felder, Kofman and Linde proposed a much more efficient mechanism of reheating (called the instant preheating) [8]. In this paper, we calculate the spectra of gravitational waves in both cases of the instant preheating and gravitational reheating, and

¹ This possibility was noted in a footnote of [1].

examine the dependence of the gravitational wave spectrum on the reheating process.

This paper is organized as follows. In Sec. II, we introduce our model of quintessential inflation. In Sec. III, we briefly review two reheating processes after quintessential inflation. In Sec. IV, we calculate the spectrum of the gravitational waves. Sec. V is devoted to conclusion. We use the units of $\hbar = c = 1$.

II. YET ANOTHER QUINTESSENTIAL INFLATION

We utilize an exponential potential of the form ($M_{\text{pl}} = 1/\sqrt{8\pi G} = 2 \times 10^{18}$ GeV)

$$V(\phi) = V_0 \exp(-\lambda \phi/M_{\text{pl}}), \quad (1)$$

but with λ being a function of ϕ ; $\lambda = \lambda(\phi)$.

Inflation. If the Universe is dominated by a scalar field, then the scale factor $a(t)$ becomes $a \propto t^{2/\lambda^2}$ and the power-law inflation is realized for $\lambda < \sqrt{2}$ [9]. Recent observations of cosmic microwave background anisotropies indicate that the spectral index of the scalar perturbation, n_s , is consistent with flat one: $n_s = 0.93 \pm 0.03$ [10]. Since $n_s - 1 = -\lambda^2$ under the slow-roll approximation, we require

$$\lambda \lesssim 0.3. \quad (2)$$

If we assume that inflation terminates at $\phi \simeq 0$ by changing the slope λ , then the amplitude of the density perturbations observed by COBE fixes the energy scale of inflation: $V_0 \simeq 10^{-12} M_{\text{pl}}^4$.

Kination. After the end of inflation, the universe is still dominated by ϕ . Inflation is followed by a kinetic energy-dominated epoch (hereafter called kination). During the kination, $\dot{\phi}^2 \sim V_0(a/a_{kd})^{-6}$ and the scalar field evolves as

$$\phi \simeq \sqrt{6} M_{\text{pl}} \ln(a/a_{kd}), \quad (3)$$

where a_{kd} is the scale factor at the end of inflation. Therefore in order for the potential energy $V \propto \exp(-\lambda\phi/M_{\text{pl}})$ not to dominate the energy density of the Universe, it is required that

$$\lambda > \sqrt{6}. \quad (4)$$

The presence of the epoch of kinetic energy domination is the main reason of enhancement of amplitude of gravitational waves in quintessential inflation models discussed below [1].

Dark Energy Domination. The Universe must eventually be dominated again by the scalar field to account for the acceleration of the Universe. This is accomplished by the termination of the domination of the kinetic energy of the scalar field and hence we require

$$\lambda < \sqrt{6}. \quad (5)$$

The scalar field stops moving when $V(\phi_X) \sim H_0^2 M_{\text{pl}}^2$ and the Universe becomes eventually dominated by the scalar field again.

From the requirements (2), (4) and (5), we find that different three slopes (λ) are needed for the stages of inflation, kination and dark-energy dominance. As an example, we consider a potential in the following form:

$$V(\phi) = V_0 \exp(-\lambda(\phi)\phi/M_{\text{pl}}), \quad (6)$$

$$\lambda(\phi) = \left(\tanh(\phi/M_{\text{pl}}) + A \right) \frac{(\phi/M_{\text{pl}})^2 + u^2}{(\phi/M_{\text{pl}})^2 + v^2}, \quad (7)$$

where A, u and v are constants. In the following analysis, we take $A = 1.1$ and arrange u and v to fix $\Omega_\phi = 0.7$ today. Inflation is terminated when $(aH)^{-1}$ is minimum. V_0 is fixed by the amplitude of density perturbation observed at COBE scale : $V^{3/2}/(\sqrt{75}\pi M_{\text{pl}}^3 V') = 1.9 \times 10^{-5}$.

Possible Scenario. We speculate on possible scenarios for realizing the above potential form. The first one is the system of the Lagrangian consisting of a non-canonical and a simple exponential potential:

$$-\frac{1}{2}Z(\Phi)^2(\nabla\Phi)^2 - V_0 \exp(-\alpha\Phi). \quad (8)$$

A non-canonical kinetic term appears in supergravity theories [11] and in higher-dimensional theories via dimensional reduction. Introducing a new variable ϕ such that $Z(\Phi)\partial_\mu\Phi = \partial_\mu\phi$ so that $\phi = \int Z(\Phi)d\Phi \equiv Y(\Phi)$, we may obtain the required form of the potential with $\lambda(\phi)\phi = \alpha Y^{-1}(\phi)$.

Another scenario is a scalar field with non-minimal coupling with gravity [12, 13]:

$$f(\phi)R - \frac{1}{2}(\nabla\phi)^2 - V(\phi). \quad (9)$$

Since the effective potential for the dynamics of the scalar field is $V(\phi)/f^2(\phi)$ rather than $V(\phi)$ [12, 13], we may realize the required form from a simple form of $V(\phi)$, although we must be careful about possible time-variation of the gravitational constant and deviations from general relativity [14].

III. REHEATING PROCESSES

Gravitational particle production. Massless particles are created due to the change of the expansion-law of the space-time. These particles can reheat the universe. Because the energy of massless particles decreases like that of radiation, the energy density of created particles eventually dominates even if it is initially negligible. The created particles thermalize the Universe in the following standard process [1]. The energy density of the created particles is $\rho_m = RH_{kd}^4(a/a_{kd})^{-4}$, where H_{kd} is the Hubble parameter at the end of inflation and $R \sim 10^{-2}$ per scalar field component [15, 16] and $R \simeq 10^{-2}N_s$ for N_s scalar fields.

The thermalization of ρ_m occurs when the interaction rate $n\sigma$ is faster than the expansion rate of the Universe H . Here the number density n and the cross section σ are written in terms of the energy of created particle $\epsilon \sim H_{kd}(a/a_{kd})^{-1}$ as $n \simeq R\epsilon^3$ and $\sigma \sim \alpha^2\epsilon^{-2}$ with $\alpha \sim 10^{-2}$. Thus the thermalization occurs at $a_{th}/a_{kd} \sim \alpha^{-1}R^{-1/2}$ and the temperature is $T_{th} = R^{3/4}\alpha H_{kd}$.

Since $\rho_m/\rho_\phi \sim RH_{kd}^2M_{pl}^{-2}(a/a_{kd})^{-2}$ during kination, the radiation dominated epoch begins at

$$a_r/a_{kd} = \sqrt{\frac{3}{R}}H_{kd}M_{pl} \sim 10^6 R^{-1/2} \left(\frac{H_{kd}}{10^{12} \text{ GeV}} \right). \quad (10)$$

From this result, we can obtain the temperature at the beginning of the radiation.

$$T_r \sim R^{3/4}H_{kd}(3M_{pl}^2)^{-1/2} \sim 10^6 R^{3/4} \left(\frac{H_{kd}}{10^{12} \text{ GeV}} \right) \text{ GeV}. \quad (11)$$

This temperature is sufficient for the beginning of the standard hot big bang model. A typical result of the time evolution is given in Fig. 1.

Instant preheating. We consider the so-called instant preheating as another reheating mechanism. Felder et al. proposed a much more efficient mechanism of reheating which works even for a potential without a minimum [7]. The mechanism is similar to preheating by parametric resonance [8]. They introduced the interaction Lagrangian, $-\frac{1}{2}g^2\phi^2\chi^2 - \frac{1}{2}\psi\bar{\psi}\chi$, where g and h are the coupling constants. After the end of inflation, the energy of the inflaton ϕ is transferred to the scalar field χ and the field χ decays to the fermions ψ .

For simplicity, we assume that inflation ends at $\phi = 0$ and the scalar field χ does not have a bare mass. Then, the effective mass of the field χ is $m_\chi = g|\phi|$. The particle creation of the field χ occurs when the adiabaticity condition is violated, $|\dot{m}_\chi| \gtrsim m_\chi^2$. This happens

right after the kination begins. Hence when

$$|\phi| \lesssim \phi_* = \sqrt{|\dot{\phi}_{kd}|/g} \simeq V_0^{1/4}/\sqrt{g}, \quad (12)$$

where $\dot{\phi}_{kd}$ is $\dot{\phi}$ at the end of inflation. In order for the particle creation to be completed within a sufficiently short period, we must have $\phi < M_{\text{pl}}$ and g must satisfy $g > 10^{-6}$. The time interval of the particle production is then written as

$$\Delta t \sim \frac{\phi_*}{|\dot{\phi}_{kd}|} \simeq g^{-1/2} V_0^{-1/4}. \quad (13)$$

Thus the created particles will have typical momenta $k \sim \Delta t^{-1} \simeq g^{1/2} V_0^{1/4}$ [7]. The occupation number of the created particles is $n(k) = \exp(-\pi k^2/g\sqrt{V_0})$ [8]. Integrating this equation, we can get the number density of the field χ

$$n_\chi = \frac{1}{2\pi^2} \int_0^\infty dk k^2 n(k) = \frac{(g\sqrt{V_0})^{3/2}}{8\pi^3}. \quad (14)$$

The energy density immediately after the creation is $\rho_\chi \sim (g\sqrt{V_0})^2/8\pi^3$. Soon after that, the field χ decays into the fermion ψ with decay rate $\Gamma = h^2 m_\chi/8\pi = h^2 g|\phi|/8\pi$. The subsequent decay of the field χ leads to a complete reheating of the Universe.

The equation of the field ϕ after particle production is written as

$$\ddot{\phi} + 3H\dot{\phi} + g^2\langle\chi^2\rangle\phi = 0, \quad (15)$$

where the potential can be neglected in the kination. When the field ϕ becomes greater than ϕ_* and particle production ends, the particles χ become nonrelativistic. Then we can calculate $\langle\chi^2\rangle$

$$\langle\chi^2\rangle \approx \frac{1}{2\pi^2} \int \frac{n_k k^2 dk}{\sqrt{k^2 + g^2\phi^2}} \approx \frac{n_\chi}{g\phi} \approx \frac{(g\sqrt{V_0})^{3/2}}{8\pi^3 g\phi} \left(\frac{a_{kd}}{a}\right)^3. \quad (16)$$

Therefore the backreaction of created particles on ϕ is negligible if $g^2\langle\chi^2\rangle\phi < 3H\dot{\phi}$, that is

$$a < a_{kd} \left(\frac{8\sqrt{6}\pi^3 V_0^{1/4}}{g^{5/2} M_{\text{pl}}} \right)^{1/3}. \quad (17)$$

We consider the case of $a_b/a_{kd} > 1$, so we get $g < 0.8$ from Eq.(17). If $\Gamma > H_b$, where H_b is the Hubble parameter at the scale factor $a_b = a_{kd}[8\sqrt{6}\pi^3 V_0^{1/4}/(g^{5/2} M_{\text{pl}})]^{1/3}$, particles χ will decay to fermions ψ at $a < a_b$ and the force driving the field ϕ back to $\phi = 0$ will disappear.

This condition is given by

$$h^2 > \frac{g^{3/2} V_0^{1/4}}{3\sqrt{2}\pi^2 M_{\text{pl}}}, \quad (18)$$

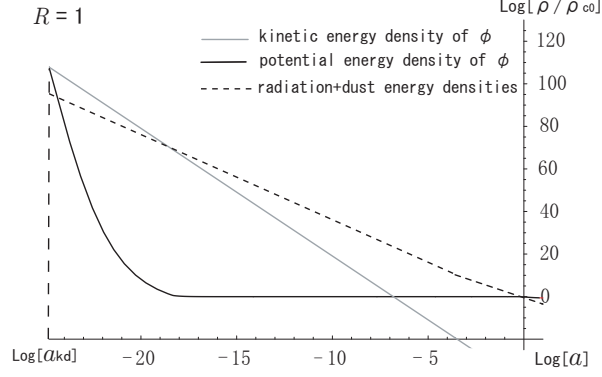


FIG. 1: Evolution of the energy densities for gravitational reheating. The solid line is the potential energy density of ϕ . The gray line is the kinetic energy density of ϕ . The dotted line is the radiation and dust energy densities.

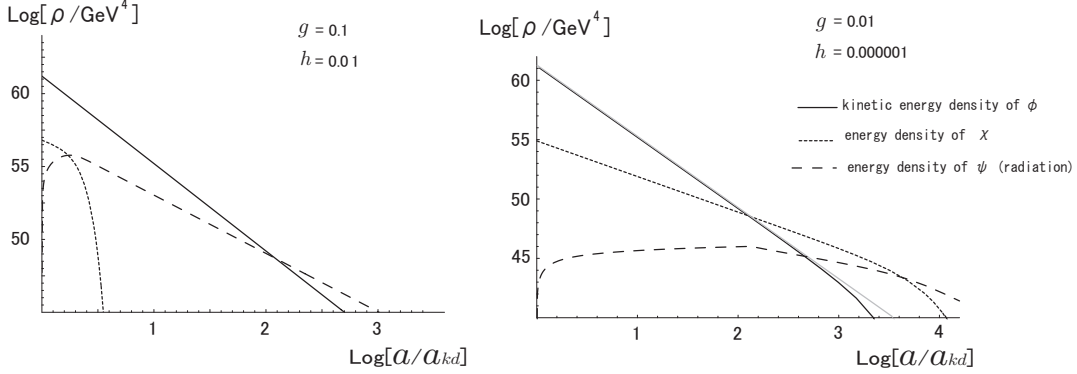


FIG. 2: Evolution of the energy densities for instant preheating. The solid line is the kinetic energy density of ϕ , the dotted line is the energy density and the dashed line is energy density of ψ . In the right panel there is the era dominated by the scalar field χ , because the decay rate of the field χ is low.

where we have assumed $\phi \sim M_{\text{pl}}$. When the backreaction is nonnegligible, the scalar field χ dominated epoch begins. The reason is that the condition (17) is essentially the same as that for $\rho_\chi < \rho_\phi$. When $H \sim \Gamma$, the field χ decays into ψ and the energy is transferred to the radiation energy. Thus the radiation dominated epoch starts. In Fig. 2, we show the results of time evolution of fields.

IV. GRAVITATIONAL WAVES

In each reheating process, we take $A = 1.1$ in Eq.(7) and arrange u, v to fix $\Omega_\phi = 0.7$ at the present. From this result, we obtain the scale factor and the Hubble parameter at the beginning of each epoch.

We calculate the graviton spectrum produced by an inflationary stage by using the Bogoliubov transformation in terms of conformal time τ [17, 18]. But the spectrum shape is easily understood in the following way before carrying out a detailed calculation. When the gravitational waves come out of the horizon during inflation, the amplitude of the gravitational waves is frozen and the value is proportional to H_{out} , where H_{out} is the Hubble parameter when the scale of the perturbation crosses the Hubble horizon. After reentering the horizon at scale factor a_{in} , the amplitude decreases as $1/a$. So the today characteristic amplitude $h_c(\omega)$ which reentered the horizon in an era with scale factor $a \propto t^n$ is

$$h_c \propto H_{out} \frac{a_{in}}{a_0} \propto f^{(3/2)-\nu+(n/n-1)}. \quad (19)$$

where

$$\nu = \frac{1}{2} \left(1 + \frac{4}{2 - \lambda^2} \right), \quad (20)$$

and λ here is its value during inflation which we approximate by $\lambda(\phi \rightarrow -\infty)$ in Eq. (6) for simplicity. This means that the scale factor during inflation is approximated to be proportional to t^{2/λ^2} . From this, the energy density of the gravitational waves per unit logarithmic interval of frequency is

$$\Omega_{gw}(f) = \frac{1}{\rho_c} \frac{d\rho_{gw}}{d \log f} = \frac{1}{6H_0^2} f^2 h_c^2(f) \propto f^{3-2\nu+(2n/n-1)+2}, \quad (21)$$

where ρ_c is the critical energy density. Our model consists of the five phases: inflationary phase (IN), kinetic-energy dominated phase (KD), radiation dominated phase (RD), matter dominated phase (MD) and potential dominated phase (PD). Among them, the phases when the gravitational waves reenter the horizon are KD ($n = 1/3$), RD ($n = 1/2$) and MD ($n = 2/3$). Correspondingly, the spectrum has three branches which are proportional to $f^{1+(3-2\nu)}$, $f^{3-2\nu}$ and $f^{-2+(3-2\nu)}$ and the shapes are showed in Fig. 3.

The detailed calculation using the Bogoliubov transformation is as follows. The scale factor in each phase is given by

$$a(\tau) = a_{kd} \left(\frac{\tau}{\tau_{kd}} \right)^{(1-2\nu)/2} \quad \text{for IN,}$$

$$\begin{aligned}
a(\tau) &= a_{kd} \left[(1 - 2\nu) \left(\frac{\tau}{\tau_{kd}} + \frac{2\nu}{1 - 2\nu} \right) \right]^{1/2} && \text{for KD,} \\
a(\tau) &= \frac{a_{kd}^2}{a_r} \frac{1 - 2\nu}{2} \frac{\tau_r}{\tau_{kd}} \left(\frac{\tau}{\tau_r} + \frac{a_r^2}{a_{kd}^2} \frac{2}{1 - 2\nu} \frac{\tau_{kd}}{\tau_r} - 1 \right) && \text{for RD,} \\
a(\tau) &= a_d \left(\frac{1 - 2\nu}{4} \frac{a_{kd}^2}{a_d a_r} \frac{\tau_d}{\tau_{kd}} \right)^2 \left[\frac{\tau}{\tau_d} + \frac{4}{1 - 2\nu} \frac{a_d a_r}{a_{kd}^2} \frac{\tau_{kd}}{\tau_d} - 1 \right]^2 && \text{for MD,} \\
a(\tau) &= \frac{a_p}{2} \frac{\sqrt{a_p}}{\sqrt{a_p} - \sqrt{a_d}} \left(1 - \frac{\tau_d}{\tau_p} \right) \left[\frac{1}{2} \frac{\sqrt{a_p}}{\sqrt{a_p} - \sqrt{a_d}} \left(1 - \frac{\tau_d}{\tau_p} \right) + 1 - \frac{\tau}{\tau_p} \right]^{-1} && \text{for PD,} \quad (22)
\end{aligned}$$

where subscripts kd , r , d and p mean the epochs of the phase transitions between IN-KD, KD-RD, RD-MD and MD-PD stages, respectively. In the PD phase, we have assumed that the universe accelerates exponentially, since the potential energy density is nearly constant as seen from Fig. 1. The conformal time at each phase transition is expressed as

$$\begin{aligned}
\tau_{kd} &= \frac{1 - 2\nu}{2} \frac{1}{a_{kd} H_{kd}}, \\
\tau_r &= \left(\frac{a_r^2}{a_{kd}^2} - 1 \right) \frac{1}{2a_{kd} H_{kd}} + \tau_{kd}, \\
\tau_d &= \left(\frac{a_d}{a_r} - 1 \right) \frac{1}{a_r H_r} + \tau_r, \\
\tau_p &= \left(\sqrt{\frac{a_p}{a_d}} - 1 \right) \frac{2}{a_d H_d} + \tau_d. \quad (23)
\end{aligned}$$

Note that $\tau_{kd} < 0$.

In the TT (transverse-traceless) gauge, a positive frequency function for the gravitational perturbation with comoving wave number \mathbf{k} is represented by

$$h_{ij}^{(\sigma, \mathbf{k})} = a(\tau)^2 e_{ij}^{(\sigma)}(\mathbf{k}) \mu_k(\tau) e^{i\mathbf{k} \cdot \mathbf{x}}, \quad (24)$$

where $e_{ij}^{(\sigma)}(\mathbf{k})$ ($\sigma = 1, 2$) are two independent polarization tensors normalized as

$$e^{(\sigma)ij}(\mathbf{k}) e_{ij}^{(\sigma')}(\mathbf{k}) = \delta^{\sigma\sigma'}. \quad (25)$$

The Einstein equations give an equation for μ_k

$$\mu_k'' + 2 \frac{a'}{a} \mu_k' + k^2 \mu_k = 0, \quad (26)$$

where the prime denotes the derivative with respect to the conformal time. The mode function μ satisfies the normalization

$$\mu^* \mu' - \mu \mu'^* = -i/a^2. \quad (27)$$

In the inflationary phase, assuming the standard Bunch-Davies vacuum, the solution is given by

$$\mu_i(\tau) = \frac{\sqrt{\pi}}{2a_{kd}}(-\tau_{kd})^{\frac{1}{2}-\nu}(-\tau)^\nu H_\nu^{(1)}(-k\tau), \quad (28)$$

where the subscript i means it is the solution in the inflation phase and $H_\nu^{(1)}(x)$ is the Hankel function of the first kind.

The solution in the kination is

$$\mu_{kd}(\tau) = \alpha_{kd}\psi_{kd}(\tau) + \beta\psi_{kd}^*(\tau) \quad (29)$$

$$\psi_{kd}(\tau) = \frac{\sqrt{\pi}}{2a_{kd}} \left(\frac{1}{1-2\nu} \tau_{kd} \right)^{1/2} H_0^{(2)} \left(k \left[\tau + \frac{2\nu}{1-2\nu} \tau_{kd} \right] \right), \quad (30)$$

where α_{kd} and β_{kd} are called the Bogoliubov coefficients, ψ_{kd} is the mode function in the kination normalized as Eq. (27), and $H_0^{(2)}(x)$ is the Hankel function of the second kind. The Bogoliubov coefficients are determined by the condition that μ and μ' be continuous at the phase transition. From this condition at $\tau = \tau_{kd}$ we get

$$\begin{aligned} \alpha_{kd} &= \frac{1}{H_\nu^{(1)}(-k\tau_{kd})} \left[\left(\frac{1}{1-2\nu} \right)^{1/2} H_\nu^{(1)}(-k\tau_{kd}) - \beta_k H_0^{(1)} \left(\frac{1}{1-2\nu} k\tau_{kd} \right) \right], \\ \beta_{kd} &= \frac{i\pi}{8} \left(\frac{4}{1-2\nu} \right)^{1/2} \tau_{kd} \\ &\times \left[-\frac{k}{2} H_1^{(2)} \left(\frac{1}{1-2\nu} k\tau_{kd} \right) H_\nu^{(1)}(-k\tau_{kd}) - \nu \tau_{kd}^{-1} H_\nu^{(1)}(-k\tau_{kd}) H_0^{(2)} \left(\frac{1}{1-2\nu} k\tau_{kd} \right) \right. \\ &\left. + \frac{k}{2} H_0^{(2)} \left(\frac{1}{1-2\nu} k\tau_{kd} \right) \left(H_{\nu-1}^{(1)}(-k\tau_{kd}) - H_{\nu+1}^{(1)}(-k\tau_{kd}) \right) \right]. \end{aligned} \quad (31)$$

For wavelengths of cosmological interest, we have $-k\tau_{kd} \ll 1$. When we assume $-k\tau_{kd} \ll 1$, we get

$$\beta_{kd} = \frac{\Gamma(\nu)}{\pi} \left(\frac{1-2\nu}{4} \right)^{1/2} \left[\left(\frac{4}{1-2\nu} \right) \frac{a_{kd} H_{kd}}{k} \right]^\nu, \quad (32)$$

where we have replaced τ_{kd} by its expression in terms of the scale factor and the Hubble parameter given in Eq.(23).

Here we need to mention the adiabatic theorem [17, 18]. The transition of the phases is not instantaneous in reality. The change of the cosmological evolution occurred with the time scale $\Delta T = H^{-1}$. The time scale associated with the mode function with the comoving wavenumber \mathbf{k} is the frequency $\omega = k/a$. If $k/a \geq \Delta T = H^{-1}$, this change of phase is adiabatic and no particle creation occurs. This results in $\alpha = 1$, $\beta = 0$ for $k \geq aH$ at an epoch of a phase transition.

In order to obtain the spectrum of the present gravitational waves we need the solution in the potential dominated epoch. It is acquired by the the Bogoliubov transformation. The relation between the mode functions are written using the Bogoliubov coefficients.

$$\begin{aligned}\psi_{kd} &= \alpha_r \psi_r + \beta_r \psi_r^*, \\ \psi_r &= \alpha_d \psi_d + \beta_d \psi_d^*, \\ \psi_d &= \alpha_p \psi_p + \beta_p \psi_p^*.\end{aligned}\tag{33}$$

These Bogoliubov coefficients are obtained by requiring the continuity condition at each phase transition. The mode function at the PD epoch is expressed as

$$\mu_p = \alpha_{total} \psi_p + \beta_{total} \psi_p^*,\tag{34}$$

where α_{total} and β_{total} are given by

$$\begin{pmatrix} \alpha_{total} & \beta_{total} \\ \beta_{total}^* & \alpha_{total}^* \end{pmatrix} = \begin{pmatrix} \alpha_{kd} & \beta_{kd} \\ \beta_{kd}^* & \alpha_{kd}^* \end{pmatrix} \begin{pmatrix} \alpha_r & \beta_r \\ \beta_r^* & \alpha_r^* \end{pmatrix} \begin{pmatrix} \alpha_d & \beta_d \\ \beta_d^* & \alpha_d^* \end{pmatrix} \begin{pmatrix} \alpha_p & \beta_p \\ \beta_p^* & \alpha_p^* \end{pmatrix}.\tag{35}$$

In the PD epoch, the wavelength cannot come into the horizon because of the accelerated expansion. So, for all scales which we can observe ($k > a_0 H_0$), we have $\alpha_p = 1$ and $\beta_p = 0$.

The number density of created gravitons with conformal wave number \mathbf{k} is

$$N(k) = |\beta_{total}|^2.\tag{36}$$

The energy density of gravitational waves with frequency $f = k/2\pi a_0$ is written taking into account of two polarization of the gravitational waves as

$$d\rho_{\text{gw}}(f) = 2 (2\pi f) |\beta_{\text{total}}|^2 4\pi f^2 df.\tag{37}$$

From this the density parameter of gravitational waves is

$$\Omega_{\text{gw}}(f) = 16\pi^2 \rho_c^{-1} |\beta_{\text{total}}|^2 f^4.\tag{38}$$

Thus we finally obtain

$$\Omega_{\text{gw}}(f) = \frac{4(1-2\nu)\Gamma^2(\nu)}{3M_{\text{pl}}^2 H_0^2} \left[\frac{2H_{kd}}{\pi(1-2\nu)} \frac{a_{kd}}{a_0} \right]^{2\nu} f^{1+(3-2\nu)}, \quad \frac{H_r}{2\pi} \frac{a_r}{a_0} < f < \frac{H_{kd}}{2\pi} \frac{a_{kd}}{a_0},\tag{39}$$

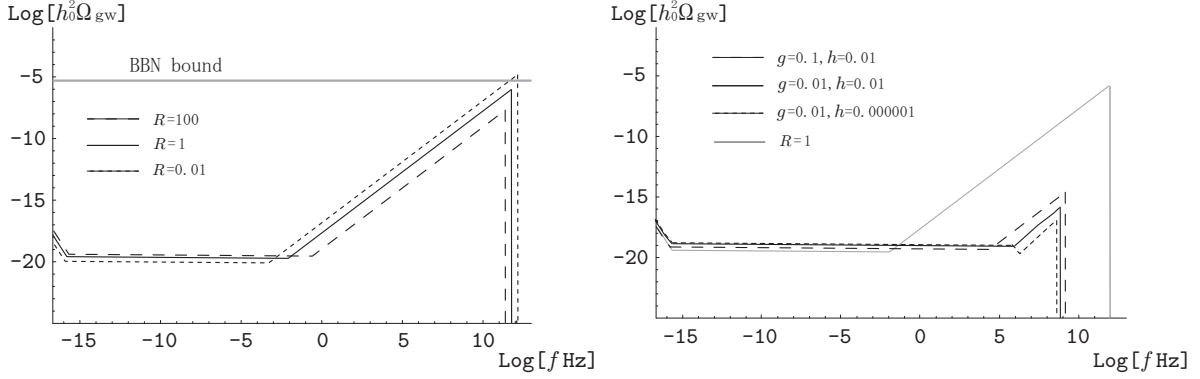


FIG. 3: The spectrum of gravitational waves. The left panel is in the case of the reheating by gravitational particle production. The right one is in the case of instant preheating. For comparison, we also plot the result of gravitational reheating with $R = 1$.

$$\Omega_{\text{gw}}(f) = \frac{2(1-2\nu)\Gamma^2(\nu)}{3\pi M_{\text{pl}}^2 H_0^2} \left[\frac{2H_{kd}}{\pi(1-2\nu)} \frac{a_{kd}}{a_0} \right]^{2\nu} H_r \frac{a_r}{a_0} f^{(3-2\nu)}, \quad \frac{H_d a_d}{2\pi a_0} < f < \frac{H_r a_r}{2\pi a_0}, \quad (40)$$

$$\Omega_{\text{gw}}(f) = \frac{(1-2\nu)\Gamma^2(\nu)}{6\pi^3 M_{\text{pl}}^2 H_0^2} \left[\frac{2H_{kd}}{\pi(1-2\nu)} \frac{a_{kd}}{a_0} \right]^{2\nu} \left(H_d \frac{a_d}{a_0} \right)^2 H_r \frac{a_r}{a_0} f^{-2+(3-2\nu)}, \quad \frac{H_0}{2\pi} < f < \frac{H_d a_d}{2\pi a_0}. \quad (41)$$

Note that $\nu = 3/2$ corresponds to de Sitter inflation.

The spectrum of gravitational waves for each reheating process is shown in Fig. 3. The effect of kination shows up in the band $a_r H_r / 2\pi a_0 < f < a_{kd} H_{kd} / 2\pi a_0$. The spectrum in this band is not flat but proportional to $f^{1+(3-2\nu)}$. Therefore the gravitational waves at high frequency are amplified much more than that in ordinary inflation models. The duration of kination depends on the efficiency of reheating. The longer the duration is, the larger the amplitude becomes. The gravitational particle production is a least efficient reheating mechanism, which may give rise to the amplitude of gravitational waves large enough to be detectable. However the energy density of the gravitational waves is constrained by the success of the Big Bang nucleosynthesis, $\Omega_{\text{gw}} < \Omega_{\text{gw}}^{BBN} = 5 \times 10^{-6} h_0^{-2}$ [19]. Hence the efficiency R of the reheating by gravitational particle production has a lower limit,

$$R \gtrsim 0.2 (\Omega_{\text{gw}}^{BBN} / 5 \times 10^{-6} h_0^{-2})^{-1} (H_{kd} / 10^{12} \text{ GeV})^4. \quad (42)$$

The sensitivity of the advanced LIGO will be $\Omega_{\text{gw}} \simeq 10^{-11}$ at $10 \sim 100 \text{ Hz}$ and that of LISA will $\Omega_{\text{gw}} \simeq 10^{-11}$ at 1 mHz [19]. Thus the detectability gravitational waves by these

experiments appears unlikely.

When we calculate the gravitational wave spectrum we assumed that ν is constant in the inflationary phase. This is because ν changes rapidly at the end of inflation in our toy model. However, if we consider a model that ν changes mildly, we must consider corresponding effects on the spectrum. If ν is larger than $3/2$ in the smaller scales than the COBE bound region ($\Omega_{\text{gw}} < 7 \times 10^{-11} (H_0/f)^2 h_0^{-2}$, $10^{-18} \text{Hz} < f < 10^{-16} \text{Hz}$ [19]), the slope in the bands of the kination becomes looser and the energy density of the gravitational waves may be high enough in the detectable bands by LIGO and LISA.

Instant preheating is so efficient that the duration of kination is much shorter than that in gravitational reheating. This results in suppression of the enhancement of the spectrum of the gravitational waves at $\sim 100 \text{MHz}$. Hence we find that the spectrum of the gravitational waves is quite sensitive to the reheating process after inflation. From Fig. 3 we may obtain information regarding the thermal history of the universe after inflation from the spectrum of the gravitational waves at $\sim 100 \text{MHz}$.

V. CONCLUSION

We investigated the possibility of quintessential inflation by an exponential-type potential. In this inflation model, the present accelerated expansion is explained by the same scalar field that caused inflation in the past. We considered two extremes of reheating process: the gravitational particle production which is least efficient and the instant preheating which is most efficient and has not been considered in the context of quintessential inflation. We calculated the spectrum of gravitational waves produced during inflation. We found that the spectrum of the gravitational waves is quite sensitive to the reheating process. Less efficient reheating results in larger amplitude of the spectrum. We emphasize that these results are not limited to quintessential inflation and insensitive to the detailed model of inflation since the spectrum is essentially determined by the expansion rate during inflation and that at the horizon-crossing time. On very large scales corresponding to the Hubble rate at an early stage of inflation, it is severely constrained by CMB observations, while on very small scales, the dominant constraint comes from BBN which is less stringent. Therefore, while it is definitely important to obtain the shape of the spectrum at high frequencies ($\sim 100 \text{ Hz}$) or at very high frequencies ($\sim 100 \text{ MHz}$), even the detection or nondetection of the

gravitational waves would provide us with useful information of the early universe.

In particular, for detectors with arm length of 1 m, for example, the typical frequency range would be 100 MHz. Such tabletop gravitational wave detectors may be constructed with much less cost [20]. The implications of such high frequency detectors on the early universe would be enormous.

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